VIII. *Account of Experiments on Torsion and Flexure for the Determination of Rigidities.*

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IN my Paper read February 22nd, 1866, the intention was expressed of continuing my experiments on rigidity with a modified form of apparatus. This intention was carried out during the past summer, and I have now to report the results.

In the former experiments, the rod operated on was supported at both ends, and was bent or twisted by hanging a pair of equal weights so as to act symmetrically on both ends; and the amounts of flexure and torsion were measured by the movements of two images formed by reflection upon a screen.

In the new apparatus, the rod was firmly held at one end in such a manner that this end could undergo no movement whatever, while the other end was acted on by a couple composed of the direct action of a weight and the upward pull of one arm of a balance produced by weighting the other arm. The effect produced was observed, as in KIRCHHOFF'S experiments, by means of two telescopes looking down into two mirrors which reflected a scale of lines crossing each other at right angles placed horizontally overhead.

A B (Plate IX. fig. 1) is the rod operated on, entering a socket in the cylindrical iron bar C, in which it is firmly secured by screws (three in each set) which clamp it at two places about 2 inches asunder. The other end A passes through a brass socket (shown in cross section at fig. 2), to which it is also secured by screws in two places. This socket forms part of a piece of brass, which is shown on a larger scale in longitudinal section in fig. 3, where *n* is a point or cone to be supported by a ring (M, fig. 1) hanging from one arm of a balance, while the lower part consists of a short cylinder *m* (for receiving the crosspiece shown in fig. 4 and indicated by dotted lines in fig. 3) terminated by a screw which receives the nut *p p*. The circular hollow shown in the centre of the cross piece (fig. 4) fits the cylinder *m*, and the crosspiece can either be rotated about it or slipped off on loosening or removing the nut. The four arms of the crosspiece are all of equal length, and each of them has on the upper side near the end a cone or point for supporting a weight by means of a ring. F F is a cast-iron box, on the top of which the cylinder C rests in two notches one at each end, in which it turns freely when not secured by the clamp G. H is a graduated circle for turning the cylinder (and with it the rod A B) through any required angle. K, L are two mirrors clamped to the rod, and adjustable by footscrews into any position nearly parallel with the rod. One of them is shown on a larger scale at fig. 5. By partially releasing the clamps, it was easy to rotate the mirrors about the rod without longitudinal sliding.

The point *n*, fig. 3, is supported by the flat brass ring M, which hangs by the wire N from one arm of the balance D, and a counterpoise is placed in the pan P just sufficient to keep the rod A B free from strain.

The experiments were conducted in the Natural Philosophy Lecture-room. The box F F rested on the floor, the height of mirrors above floor being 270 millims. The scale reflected by them consisted of a large sheet of paper ruled in two directions at right angles to each other with lines about a tenth of an inch apart, and was firmly fixed at the height of 4597 millims. from the floor by stretching it on a board and screwing this to two joists whose primary office was the support of a cistern. The light, which was naturally good, was improved by using a concave mirror to illuminate the scale. Two telescopes, not shown in the Plate, were clamped to a firm three-legged table, their object-glasses being about 970 millims. above the floor. They were in fixed positions, directed one towards each mirror, and were as nearly vertical as was compatible with an unobstructed view of the reflection of the scale. Their deviations from two vertical planes, one parallel and the other perpendicular to the rod, were from $\frac{1}{54}$ to $\frac{1}{20}$ in circular measure. They were inverting achromatic, of $1\frac{1}{8}$-inch aperture and 10 inches focal length, with cross wires in focus of eyepiece. A damper, consisting of a piece of thin card pressing lightly against the end A of the rod, was used on and after July 17th for the purpose of checking vibration.

The mode of observing for flexure was as follows:—The mirrors having been adjusted so as to bring the central portion of the scale into view in both telescopes, a pair of equal weights were placed, one in the scale-pan P along with the counterpoise, the other on the point S, and readings were taken in both telescopes. Then the weight at S was transferred to S′, and readings were again taken. The difference of readings in further telescope diminished by difference of readings in nearer telescope is assumed to measure the effect, on the portion of rod between the two mirrors, of a bending couple whose arm is the distance between the two points S, S′, and whose power is the force of gravity on the moveable weight.

The weight was then transferred first to T and then to T′, both telescopes being read in each case. The differences were taken in the same way as above, and the result is assumed to measure the effect, on the same portion of the rod, of a twisting couple whose power is the same as above, and whose arm is the distance between the points T, T′.

The weight was then again transferred to S′ and S, then again to T′ and T, and so on several times, both telescopes being read in each position of the weight, and no change being made in any of the adjustments. The facility of thus passing from observations of flexure to those of torsion, and *vice versâ*, gives the present form of apparatus a great superiority over that employed the previous year.

It has been observed that the arms of couple in flexure and torsion are the distances S S′, T T′ respectively, which, though nearly equal, are not absolutely identical. This defect was easily remedied by turning the crosspiece through a right angle, so as to make $S S'$ change places with $T T'$.

Another source of error to be guarded against is want of perfect circularity in the rod operated upon. This is completely removed, if the deviation from circularity be small, by turning the rod itself through a right angle by means of the graduated circle H. This change has no effect on the torsional rigidity; and its effect on the flexural rigidity is such that the mean flexure in the two positions is the true mean for all positions, inasmuch as the flexural rigidity in any position is proportional to the moment of inertia of a section about a horizontal diameter through its centre of gravity, and by a well-known theorem the sum of the moments of inertia about two rectangular diameters is constant.

For greater security the rod was turned into six different positions, differing by 30° among themselves, so that the first and fourth positions furnished one mean, the second and fifth another, and the third and sixth another. In every one of the six positions observations of both flexure and torsion were taken; and the operation of turning the crosspiece through a right angle so as to make the arms of couple for flexure and torsion change places, occurred between the third and fourth positions.

The first rod experimented on, after much time spent in preliminary arrangements, was a flint-glass rod from the works of A. and R. COCHRAN, Glasgow. The weights employed for producing flexure and torsion were a pair of lead weights of 100 grms. each. One of them (distinguishable by its ring) was hung in turn on each of the four arms, and the other was always placed in the counterpoise pan.

The first complete set of observations in six positions were made July 17th and 18th, with the following results:—

1 (*a*).	Pointer at 135°	Torsion 539	Flexure $435\frac{1}{3}$
2 (*a*).	„ 165°	„ $547\frac{1}{2}$	„ 438
3 (*a*).	„ 195°	„ 546	„ $446\frac{1}{2}$
1 (*b*).	„ 225°	„ 548	„ 454
2 (*b*).	„ 255°	„ $549\frac{1}{2}$	„ 454
3 (*b*).	„ 285°	„ 546	„ $447\frac{2}{3}$.

The numbers here given as representing the amounts of torsion and flexure, are expressed in tenth parts of the scale-divisions, and are therefore approximately hundredths of an inch. Combining those positions which are mutually at right angles, we have the following means:—

1 (*a*) (*b*).	Torsion $543\frac{1}{2}$	Flexure $444\frac{2}{3}$
2 (*a*) (*b*).	„ $548\frac{1}{2}$	„ 446
3 (*a*) (*b*).	„ 546	„ $447\frac{1}{2}$.

The scale-divisions were somewhat longer in one direction than in the other, being $\frac{188}{75}$ of a millimetre for torsion and $\frac{189}{75}$ millims. for flexure. In order, then, to find the true ratio of torsion to flexure, we must divide the numbers in the first column by those in the second, and diminish the quotients by $\frac{1}{189}$ of their amounts. The quotients thus corrected are

$$1\,(a)\,(b),\ 1\cdot222;\quad 2\,(a)\,(b),\ 1\cdot230;\quad 3\,(a)\,(b),\ 1\cdot221,$$

whence we obtain at once for POISSON'S ratio (σ) the values ·222, ·230, ·221. Some small corrections will be applied to these values hereafter, only affecting the third decimal place; but we deem it important thus early to direct attention to the strength of evidence showing that POISSON'S ratio for the substance in hand is less than $\frac{1}{4}$.

An earlier set of observations, in only four positions of the rod, were taken July 13th, 14th, and 16th, the apparatus being at this time less favourably arranged, inasmuch as the rod was more distant from a vertical through the centre of the scale than in the later set. The following were the results obtained:—

I (*a*).	Pointer at 90°	Torsion $555\frac{1}{3}$	Flexure 452
I (*b*).	„ 0°	„ 550	„ 430
II (*a*).	„ 45°	„ 550	„ $459\frac{1}{3}$
II (*b*).	„ 135°	„ $544\frac{1}{4}$	„ $437\frac{2}{3}$

Giving the following means,

I (*a*) (*b*).	Torsion $552\frac{2}{3}$	Flexure 441
II (*a*) (*b*).	„ $547\frac{1}{8}$	„ $448\frac{1}{2}$,

whence we obtain, after correcting for inequality of divisions, the values of POISSON'S ratio ·246, ·220, the largeness of the former number being due to the large angle made by the rays of light with the vertical plane containing the rod. A correction for this defect will be applied in the sequel.

After the observations of July 17th and 18th, the rod was removed from its place, and cut at the places where the mirrors had been attached. The length of the central portion was found to be 235·6 millims., and its weights in air and water respectively 32·002 and 21·112 grms., the temperature of the water being 13·3 Reaum.

The distances S S′, T T′ were 558·2 and 557·2 millims., so that the mean arm of couple was 557·7 millims., the force being the weight of 100 grms.

The height of the scale above the mirrors was 4327 millims.; but since the deviation of a reflected ray is double of the angle turned by mirror, it will be necessary to divide the arcs traversed on the scale by twice this distance, or 8654 millims., in order to find the angles turned.

The scale-divisions for torsion were $\frac{188}{75}$ millim., but as they were subdivided by estimation to tenths, and it is in these tenths that the above torsion-numbers are expressed, the unit is to be regarded as the $\frac{188}{750}$ of a millimetre. In like manner the unit for the flexure-numbers is the $\frac{189}{750}$ of a millimetre. We shall denote the torsion-numbers and flexure-numbers, expressed in these units, by the letters T and F.

From the observations of July 17th and 18th we have the mean values T=546, F=446·1, which, reduced to centimetres, are 13·68 and 11·24. The whole torsion and flexure in the portion of the rod between mirrors are therefore

$$\tfrac{13\cdot 68}{865\cdot 4}=\cdot 0158 \text{ nearly in circular measure,}$$
$$\text{and } \tfrac{11\cdot 24}{865\cdot 4}=\cdot 0130 \quad ,, \quad ,, \quad ,,$$

We shall now investigate the corrections which must be applied to the above results.

There is, in the first place, a mechanical correction depending on the fact that the plane which contains the four points S, S′, T, T′, and which also happens to contain the centre of gravity of the bending apparatus (*i. e.* of the crosspiece and other pieces rigidly attached to it), does not contain the point n on which the apparatus is supported. Let a denote the distance of this plane below the point n, and W the weight of the bending apparatus. Also let A denote the horizontal distance of one of the points S or T from n, and w the weight hung at S or T, and let θ denote the angle through which the end of the rod is bent or twisted. Then the couple which produces bending or twisting is $w(\mathrm{A}-a\theta)$, and this is resisted by two couples, $\mathrm{W}a\theta$, due to the weight of the bending apparatus, and $t\theta$ or $f\theta$, due to the torsional or flexural rigidity t or f. We have therefore, for torsion, $w(\mathrm{A}-a\theta)=t\theta+\mathrm{W}a\theta$, whence $t=\frac{w\mathrm{A}}{\theta}-(\mathrm{W}+w)a$. The first term, $\frac{w\mathrm{A}}{\theta}$, is the uncorrected value of t, and we see that it requires a subtractive correction which bears to its whole amount the ratio $\frac{(\mathrm{W}+w)a\theta}{w\mathrm{A}}$. Hence T, being proportional to the reciprocal of t, requires an additive correction bearing the above ratio to its whole amount. The correction for F is expressed by the same formula, θ having, however, a different value.

In the present case we have, in grammes and centimetres, W=373, w=100, A=27·9, a=4·3, hence $\frac{(\mathrm{W}+w)a}{w\mathrm{A}}=\cdot 729$. Again, since the whole length of rod subjected to torsion and flexure was about 42·8, whereas the portion between the mirrors was only about 23·6, we have

$$\text{For torsion, } \theta=\tfrac{428}{236}\times\cdot 0158=\cdot 0286,$$
$$\text{For flexure, } \theta=\tfrac{428}{236}\times\cdot 0130=\cdot 0236,$$

and the products of these values of θ by ·729 are ·0208 and ·0172.

T and F therefore require the additive corrections ·0208 T and ·0172 F.

There are also two optical corrections to be considered, viz.,

1st. Correction for obliquity of ray from scale to mirror. Let β denote this obliquity, that is to say, the angle which the projection of the ray on a vertical plane perpendicular or parallel to the rod, according as we are dealing with torsion or flexure, makes with a vertical line. Then the indicated distances on the scale are always too great in the ratio of $1:1+\beta^2$. If the angles through which the two mirrors are turned are in the ratio of $m_1:m_2$, m_2 being the greater, and if the corresponding values of β are β_1 and β_2 respectively, the observed values of T and F will be too great in the ratio of $1:1+\frac{m_2\beta_2^2-m_1\beta_1^2}{m_2-m_1}$.

In the present case the ratio $m_1 : m_2$ is about 1 : 3, and the values of β for the centre of the scale in the position occupied by the apparatus on July 17th and 18th were,

$$\text{For torsion, } \beta_1=\tfrac{1}{20},\ \beta_2=\tfrac{1}{20};\ \text{for flexure, } \beta_1=\tfrac{1}{54},\ \beta_2=\tfrac{1}{32}.$$

Hence we find by the above formula that

T is too great by ·0025 T,
F ,, ,, ·0013 F.

2nd. Correction for change of distance between mirror and telescope. If the mirror is moved parallel to itself to or from the telescope by the amount b, and if ϕ denote the angle between incident and reflected ray (or rather between their projections on a vertical plane perpendicular or parallel to the rod), the change produced in the scale-reading is $b\phi$.

In the present case this correction was found to be insensible.

For the total corrections applicable to the observations of July 17th and 18th, we have therefore

$$+\cdot 0208\,\mathrm{T}-\cdot 0025\,\mathrm{T}=+\cdot 0183\,\mathrm{T},$$
$$+\cdot 0172\,\mathrm{F}-\cdot 0013\,\mathrm{F}=+\cdot 0159\,\mathrm{F},$$

and the resulting correction of the quotient $\frac{\mathrm{T}}{\mathrm{F}}$ is

$$(\cdot 0183-\cdot 0159)\frac{\mathrm{T}}{\mathrm{F}}=\cdot 0024\frac{\mathrm{T}}{\mathrm{F}}.$$

This correction reduces the values of POISSON'S ratio derived from the observations of those days to ·225, ·233, ·224.

For the observations of July 13th, 14th, and 16th, the correction of F is the same as above. As regards the optical correction of T, a distinction must be made between the observations marked I (a) (b) and those marked II (a) (b). In the former, the central portion of the scale was on the cross wires of the telescopes, in the latter a portion of the scale nearly vertical over the mirrors. The optical correction for T applicable to the centre of the scale on the date in question was $-\cdot 0089$ T, and we shall apply this correction to the values I (a) (b), so that the total correction of T for these values will be

$$+\cdot 0208\,\mathrm{T}-\cdot 0089\,\mathrm{T}=+\cdot 0119\,\mathrm{T},$$

and the corresponding correction of $\frac{\mathrm{T}}{\mathrm{F}}$ will be

$$(\cdot 0119-\cdot 0159)\frac{\mathrm{T}}{\mathrm{F}}=-\cdot 004\frac{\mathrm{T}}{\mathrm{F}},$$

which reduces the value ·246 of POISSON'S ratio to ·241. To the values II (a) (b) we shall apply the same corrections as to the observations of July 17th and 18th, and the value ·220 of POISSON'S ratio is thus reduced to ·223. The corrected values of POISSON'S ratio ·225, ·233, ·224, ·241, ·223 give the mean value ·229; and it will be noted that every one of the five determinations (whether corrected or uncorrected) is less than one-fourth.

The five determinations of T and F uncorrected and corrected, are given below. The correcting factor for T is, as already shown, 1·0183, except for I (*a*) (*b*), for which it is 1·0119. The correcting factor for F is in every case 1·0159.

	Uncorrected.		Corrected.	
	T.	F.	T.	F.
1 (*a*) (*b*).	543·5	444·7	553·4	451·8
2 (*a*) (*b*).	548·5	446·0	558·5	453·1
3 (*a*) (*b*).	546·0	447·5	556·0	454·6
I (*a*) (*b*).	552·7	441·0	559·3	448·0
II (*a*) (*b*).	547·1	448·5	557·1	455·6
Mean of corrected values . .			556·9	452·6

We now proceed to deduce, as in our former paper, the values of t, f, n, M, and k, the units being the centimetre and the weight of a gramme.

For t and f, the torsional and flexural rigidities, we have the expressions

$$t = \text{twice distance} \times \text{force} \times \text{arm} \times \text{length} \times \tfrac{7500}{188} \div \text{T},$$
$$f = \text{twice distance} \times \text{force} \times \text{arm} \times \text{length} \times \tfrac{7500}{189} \div \text{F},$$

where twice distance $=865·4$, force $=100$, arm $=55·77$, length $=23·56$. Hence we have

$$\log t = 9·65670 - \log \text{T} = 6·91092,$$
$$\log f = 9·65440 - \log \text{F} = 6·99869.$$

The volume of the rod was 10·902, being the loss of weight in water multiplied by 1·00111, which is the factor proper to the temperature 13·3 R. The length being 23·56, we find (putting r for radius of rod) $\pi r^2 = ·46273$, $r = ·38378$.

For YOUNG'S modulus we have

$$\text{M} = \frac{4f}{\pi r^4} = 585{,}100{,}000\,;$$

for the rigidity,

$$n = \frac{2t}{\pi r^4} = 239{,}020{,}000\,;$$

for the resistance to compression,

$$k = \frac{\text{M}n}{3(3n - \text{M})} = 353{,}264{,}000\,;$$

for POISSON'S ratio,

$$\sigma = \frac{\text{M}}{2n} - 1 = \frac{f}{t} - 1 = \frac{\text{T}}{\text{F}} - 1 = ·229.$$

The values found last year for another specimen of flint glass, by a different maker (see former Paper), were

$$\text{M} = 614{,}330{,}000, \quad n = 244{,}170{,}000,$$
$$k = 423{,}010{,}000, \quad \sigma = ·258,$$

the specific gravity of the present specimen being 2·935, while that of last year's was 2·942.

The differences in these determinations of M and n, being only about five per cent. in the former case and two in the latter, are probably real, the denser specimen being also the more rigid. The values of k and σ are liable to a larger percentage of error; but this remark is more especially applicable to last year's results, as our present apparatus affords greatly increased facilities for determining the *ratio* of flexural to torsional rigidity.

With respect to the composition of the two specimens, I am unable to give precise information, as the ingredients are mixed according to no definite rule.

The glass rod having been taken down, a rod of drawn brass was mounted in its place, the apparatus remaining in precisely the same position as in the experiments of July 17th and 18th. The following results were furnished by the first set of observations, July 27th and 28th:—

I (a).	Pointer at 0°	Torsion 408	Flexure 276
II (a).	„ 30°	„ 406	„ $274\frac{1}{2}$
III (a).	„ 60°	„ 404	„ 275
I (b).	„ 90°	„ 404	„ 280
II (b).	„ 120°	„ 404	„ $276\frac{1}{2}$
III (b).	„ 150°	„ 407	„ $275\frac{1}{4}$

From these we obtain the following means:—

I (a) (b).	Torsion 406	Flexure 278,
II (a) (b).	„ 405	„ 275·5,
III (a) (b).	„ 405·5	„ 275·1,

whence, after correcting as before for difference of scale-divisions, we obtain for POISSON'S ratio the values ·451, ·461, ·465.

The weights used in these observations were the same as for the glass rod.

A second set of observations were made July 31st, August 1st and 2nd, in which, besides the old weights, which were each 100 grms., weights of 200 grms. were also employed. These latter, however, could only be used for flexure, as when the attempt was made to employ them for torsion, it was found impossible to prevent the rod from turning in its socket. In consequence of turning which took place from this cause at the commencement of this set of observations, the following pointer-readings are not precisely comparable with the foregoing, that is to say, the zero-point may be regarded as having shifted between the two sets of observations. A slight change was also made in the position of one of the telescopes, between observations 3 (a) and 1 (b), for the purpose of obtaining better light, and at the same time a string was attached to the "damper" in such a manner that the observer could pull the damper away from the rod without removing his eye from the telescope.

The following were the results, the two sets of flexure-numbers being obtained with weights of 100 and 200 grms. respectively.

1 (*a*).	Pointer at 165°	Torsion $406\frac{1}{4}$	Flexure 277,	$550\frac{1}{2}$
2 (*a*).	,, 195°	,, 406	,, 276,	551
3 (*a*).	,, 225°	,, $409\frac{1}{2}$	,, 272,	547
1 (*b*).	,, 255°	,, 410	,, 276,	552
2 (*b*).	,, 285°	,, 408	,, 279,	554
3 (*b*).	,, 315°	,, $404\frac{1}{2}$	,, 282,	$552\frac{1}{2}$

From these we have the following means:—

1 (*a*) (*b*).	Torsion 408·1	Flexure 276·5,	551·2
2 (*a*) (*b*).	,, 407·0	,, 277·5,	552·5
3 (*a*) (*b*).	,, 407·0	,, 277·0,	549·8

Correcting for difference of scale-divisions, we derive the following values of POISSON'S ratio.

From torsion at 100 grms. compared with flexure at 100 grms.,

·468, ·459, ·461.

From torsion at 100 grms. compared with flexure at 200 grms.,

·473, ·465, ·473.

Collecting all the results obtained with the brass rod, we find the mean value of T to be 406·4.

The mean value of F from the six results for weights of 100 grms. is 276·6, and from the three results for weights of 200 grms. 551·2. We shall denote these two numbers by F_1 and F_2 respectively.

Reduced to centimetres, these become

$$T\times\tfrac{188}{7500}=10{\cdot}19,\quad F_1\times\tfrac{189}{7500}=6{\cdot}97,\quad F_2\times\tfrac{189}{7500}=13{\cdot}90,$$

which, being divided by 865·4 or twice distance of scale from mirrors, give as the amounts of torsion and flexure in circular measure,

Torsion, ·01178; Flexure, ·00805 and ·0161.

The whole length of rod operated on was in the present case $\frac{477}{245}$ of the portion between mirrors; hence the values of θ for the mechanical correction are $\frac{477}{245}$ of the above angles, or

·0229, ·0157, ·0313.

The values of the factor $\dfrac{(W+w)a}{wA}$ are respectively

·729, ·729, ·442,

giving as the values of the mechanical correction

$+{\cdot}0167\ T$, $+{\cdot}0114\ F_1$, $+{\cdot}0138\ F_2$.

The first optical correction is the same as for July 17th and 18th, viz.

$-{\cdot}0025\ T$, $-{\cdot}0013\ F_1$, $-{\cdot}0013\ F_2$, and the second optical correction is still inappreciable. We have therefore as the total corrections to be applied

$$+{\cdot}0142\ T, \qquad +{\cdot}0101\ F_1, \qquad +{\cdot}0125\ F_2,$$

from which we deduce for $\frac{T}{F_1}$ and $\frac{T}{\frac{1}{2}F_2}$ the corrections $+{\cdot}0041\frac{T}{F_1}$ and $+{\cdot}0017\frac{T}{\frac{1}{2}F_2}$.

Hence the corrected values of POISSON'S ratio are—

From T and F_1 . . . ·457, ·467, ·471, ·474, ·465, ·467,
From T and F_2 . . . ·476, ·467, ·476.

The mean of these nine values is ·469, which we therefore adopt as the value of σ for brass, being nearly double of our value for glass. The comparison of our results for these two substances with those of other experimenters is somewhat startling. It stands thus:

Glass.	WERTHEIM, ·33,	MAXWELL, ·332,	EVERETT, ·239,
Brass.		KIRCHHOFF, ·387,	„ ·469;

and our two results, ·239 and ·469, were obtained with the same apparatus in the same position, each of them being the mean of several determinations, which for glass ranged from ·223 to ·241, and for brass from ·457 to ·476.

The following are the values, uncorrected and corrected, of T and F, the latter including both F_1 and $\frac{1}{2}F_2$.

	Uncorrected.		Corrected.	
	T.	F.	T.	F.
I (*a*) (*b*).	406·0	278·0	411·8	280·8
II (*a*) (*b*).	405·0	275·5	410·7	278·3
III (*a*) (*b*).	405·5	275·1	411·3	277·9
1 (*a*) (*b*).	408·1	276·5	413·9	279·3
2 (*a*) (*b*).	407·0	277·5	412·8	280·3
3 (*a*) (*b*).	407·0	277·0	412·8	279·8
1 (*a*) (*b*).	$\frac{1}{2}F_2$	275·6		279·0 $\frac{1}{2}F_2$
2 (*a*) (*b*).	$\frac{1}{2}F_2$	276·2		279·7 $\frac{1}{2}F_2$
3 (*a*) (*b*).	$\frac{1}{2}F_2$	274·9		278·3 $\frac{1}{2}F_2$
Means of corrected values . .			412·2	279·3

The elements for deriving t from T, and f from F, are the same as for the glass rod, except that the length between mirrors is 24·54 instead of 23·56.

We thus find

$$\log t = 9{\cdot}67439 - \log T = 7{\cdot}05928,$$

$$\log f = 9{\cdot}67209 - \log F = 7{\cdot}22602.$$

To determine the radius r of the rod, we have weight in air$=$91·361, weight in water $=$80·578, the temperature of the water being 7·3 R. Hence volume in centimetres $=10{\cdot}783\times1{\cdot}0002=10{\cdot}785$, which, being divided by the length 24·54, gives $\pi r^2={\cdot}43949$.

Hence we find for brass,

$$M=\frac{4f}{\pi r^4}=1,094,800,000,$$

$$n=\frac{2t}{\pi r^4}=\ 372,890,000,$$

$$k=\frac{Mn}{3(3n-M)}=5,700,700,000,$$

$$\sigma=\frac{M}{2n}-1=\cdot 469.$$

From comparing the above value of k with its values for the two glass rods experimented on, it would appear that brass is from $13\frac{1}{2}$ to 16 times more incompressible than glass; but this result is to be received with caution, for reasons which will be stated further on.

A rod of cast steel was next operated on, with the following results, the weights used being the same as for the brass rod.

I (*a*).	Pointer at 310°	Torsion $204\frac{1}{3}$	Flexure $155\frac{1}{2}$, 313
II (*a*).	„ 280°	„ 205	„ 155 , 307
III (*a*).	„ 250°	„ 207	„ 154 , 316
I (*b*).	„ 220°	„ 206	„ 157 , 313
II (*b*).	„ 190°	„ $206\frac{4}{5}$	„ 154 , $313\frac{1}{3}$
III (*b*).	„ 160°	„ $206\frac{6}{7}$	„ 156 , $313\frac{1}{3}$

Hence we have the following means:—

I (*a*) (*b*).	Torsion 205·1	Flexure 156·2, 313·0
II (*a*) (*b*).	„ 205·9	„ 154·5, 310·1
III (*a*) (*b*).	„ 206·9	„ 155·0, 314·7

Correcting for difference of scale-divisions, we obtain the following determinations of POISSON'S ratio.

From torsion at 100 grms. compared with flexure at 100 grms.,

·305, ·325, ·327.

From torsion at 100 grms. compared with flexure at 200 grms.,

·304, ·321, ·308.

As the apparatus was disturbed in my absence, and the mirrors were moved from their places before any measurements had been made of their positions, it is impossible to determine with accuracy from the foregoing observations the torsional and flexural rigidities of the rod. In order to determine POISSON'S ratio as accurately as the data permit, we shall assume (what is known to be near the truth) that the ratio of the whole length operated on to the length between mirrors was the same as for the brass rod, and that the optical corrections are the same. From these data, the mechanical corrections are found to be

$$+\cdot 0084\ T, \qquad +\cdot 0064\ F_1, \qquad +\cdot 0078\ F_2,$$

which, together with the optical corrections

$$-\cdot 0025\ T, \qquad -\cdot 0013\ F_1, \qquad -\cdot 0013\ F_2,$$

make the total corrections

$$+\cdot 0059\ T, \qquad +\cdot 0051\ F_1, \qquad +\cdot 0065\ F_2,$$

which are so small and so nearly equal that the corrections of $\frac{T}{F}$ may be neglected. We therefore assume as the value of POISSON'S ratio from these experiments, the mean of the six determinations above given, which is ·315.

All the foregoing experiments were conducted by myself in the Lecture-room during the Vacation. At the commencement of the Session, the apparatus was removed to another room, where experiments on the steel rod were continued, under my direction, by two students (Messrs. KING and WALKER) during the months of November and December. The room selected for this purpose being on the ground-floor, and paved with asphalt, on which the apparatus rested, was superior, as regards steadiness, to the lecture-room, which is on the first floor; and I may here remark that the inconsistencies (such as they are) which occur in the foregoing experiments, were found to be due mainly to the yielding of the floor under the feet of the observer.

On the other hand the new situation afforded less height, the scale being only 223·5 centimetres above the mirrors. It was also rather dark; but this defect was completely remedied by using a gaslight, aided by a concave reflector, to illuminate the scale. The scale used was a new one, of the same kind as the old, but with the lines nearer together, their distances, as determined by taking the means of several measurements, being such that

For torsion . . 171 scale-divisions =23·88 centims.
For flexure . . 171 scale-divisions =23·97 centims.

The whole length of rod subjected to torsion and flexure was 46·8, and the mirrors were attached at a greater distance apart than in any of the foregoing experiments, viz. 38·15 centims. The telescopes were at the same height above the mirrors as before, being clamped to the same table which had been previously used. The weights employed were of 100 grms., and the system of observing was the same as in the later observations above described.

The following were the values obtained for T and F in terms of their respective scale-divisions, each of these values being the mean of sixteen determinations.

1 (*a*).	Pointer at	0°	Torsion 25·62	Flexure 19·68
2 (*a*).	,,	30°	,, 25·87	,, 19·57
3 (*a*).	,,	60°	,, 25·87	,, 19·64
1 (*b*).	,,	90°	,, 25·95	,, 19·74
2 (*b*).	,,	120°	,, 25·85	,, 19·82
3 (*b*).	,,	150°	,, 25·84	,, 19·80

Hence we have the following means:—

1 (*a*) (*b*).	Torsion 25·78	Flexure 19·71
2 (*a*) (*b*).	" 25·86	" 19·70
3 (*a*) (*b*).	" 25·86	" 19·72

And applying the correction for difference of scale-divisions, which is now 1 part in 266 to be subtracted from $\frac{T}{F}$, we have as the values of POISSON'S ratio, or $\frac{T}{F}-1$,

·303, ·308, ·306, giving a mean of ·306.

The mean values of T and F are respectively 25·83 and 19·71, which reduced to centimetres become 3·61 and 2·76; and as twice the height of the scale is 447, we find the amounts of torsion and flexure respectively in the portion of rod between mirrors, to be about ·00808 and ·00617. The values of θ are $\frac{468}{381}$ of these, or ·00993 and ·00758, which are to be multiplied by ·729, as before, giving for the mechanical corrections the values

$$+\cdot 0072 \text{ T and } +\cdot 0055 \text{ F}.$$

No measurements were made to determine the optical corrections, we shall therefore assume them to be the same as in the experiments on the brass rod, viz.

$$-\cdot 0025 \text{ T and } -\cdot 0013 \text{ F},$$

making the total corrections

$$+\cdot 0047 \text{ T and } +\cdot 0042 \text{ F},$$

whose difference is so small that the correction for $\frac{T}{F}$ may be neglected. We therefore adopt for POISSON'S ratio, as determined by these experiments, the above value ·306.

The corrected mean values of T and F are 25·95 and 19·79, and we have

$$t=447\cdot 0\times 100\times 55\cdot 77\times 38\cdot 15\times \tfrac{171}{23\cdot 88}\div \text{T},$$
$$f=447\cdot 0\times 100\times 55\cdot 77\times 38\cdot 15\times \tfrac{171}{23\cdot 97}\div \text{F},$$

whence

$$\log t=9\cdot 83317-\log \text{T}=7\cdot 41903,$$
$$\log f=9\cdot 83153-\log \text{F}=7\cdot 53508.$$

The weights in air and water were respectively 132·94 and 116·00 grms., the temperature of the water being 7·7 R., and the length of the portion weighed being 38·1 centims. The correction for density at this temperature may be neglected, and we have volume in centim. = loss of weight in grammes $=16\cdot 94$. Hence $\pi r^2=\frac{16\cdot 94}{38\cdot 1}=\cdot 44462$ $r=\cdot 37620$.

$$\text{M}=\frac{4f}{\pi r^4}=2{,}179{,}300{,}000,$$

$$n=\frac{2t}{\pi r^4}=\ 834{,}120{,}000,$$

$$k=\frac{\text{M}n}{3(3n-\text{M})}=1{,}875{,}600{,}000,$$

$$\sigma=\frac{\text{M}}{2n}-1=\cdot 306, \text{ as above;}$$

and as the experiments in the Lecture-room gave ·315 as the value of σ, I adopt the value ·310. KIRCHHOFF'S value for steel is ·294, and CLERK MAXWELL'S for iron ·267.

The following are the collected results of the experiments described in both this and the former paper, the values of M, n, and k being reduced to kilogrammes' weight per square millimetre.

	Flint Glass, 1865.	Flint Glass, 1866.	Drawn Brass, 1866.	Cast Steel, 1866.
M	6143	5851	10948	21793
n	2442	2390	3729	8341
k	4230	3533	57007	18756
σ	·258	·229	·469	·310
Specific gravity .	2·942	2·935	8·471	7·849

Strictly speaking, the above values of M are the measures of resistance to longitudinal extension *parallel to the length* of the rods, and the above values of n are the measures of resistance to shearing in planes *parallel or perpendicular to the length.* The values of k and σ have been deduced on the hypothesis that the materials of the rods are isotropic. If, however, as is probably the case, this hypothesis is not fulfilled, and if the deviation from isotropy be such that the resistance to shearing in planes parallel or perpendicular to the length is less than for intermediate planes, then the values of k and σ above calculated are too large; for longitudinal extension (especially if accompanied by lateral contraction) involves a certain amount of shearing in planes oblique to the length, and the resistance to this shearing is one of the constituents of M, whereas the shearing which takes place in torsion is perpendicular to the length. Such a deviation from isotropy as we are now considering will therefore increase the ratio of M to n, and will therefore increase σ, which is equal to $\frac{M}{2n}-1$. It will also increase k, since the value of k may be written $\frac{M}{3(1-2\sigma)}$. This caution is specially important in the case of the brass rod, both because the operation of "drawing" appears likely to produce such a deviation from isotropy as we have been describing, and also because the value of σ for this rod comes out so nearly equal to $\frac{1}{2}$ that the factor $1-2\sigma$ in the denominator of k will be greatly affected by small errors in the value of σ. For these reasons we are not disposed to attach much weight to the very large value of k which we have found for brass.

We append for comparison some of the principal results obtained by previous experimenters.

The values obtained by WERTHEIM for different specimens of glass (crystal) were,—

M 3481 to 4429, mean 4039,
n 1288 to 1687, mean 1518,
k 3569 to 4476, mean 3968;

and for different specimens of brass,

M 9665 to 10645, mean 10054,
n 3600 to 3973, mean 3745,
k 10216 to 11058, mean 10631.

SAVART'S experiments on the torsion of brass wire lead to the result $n=3682$.

KUPFFER'S values of M for nine different specimens of brass range from 8112 to 11617, the value generally increasing with the specific gravity, and the two specimens which agree most nearly with our own in specific gravity show the following results:—

Specific gravity 8·4465. Value of M 10783
Specific gravity 8·4930. Value of M 11421.

The values of M found by the same experimenter for steel range from 20569 to 21842.

The values of σ found by KIRCHHOFF, WERTHEIM, and MAXWELL have already been given. They all differ widely from our own except in the case of steel.

In conclusion we may state that, as our present form of apparatus is found extremely convenient, it is intended to use it for continuing the series of experiments which have been begun, with, however, an important modification, which will be made for the purpose of diminishing or removing the "mechanical correction."

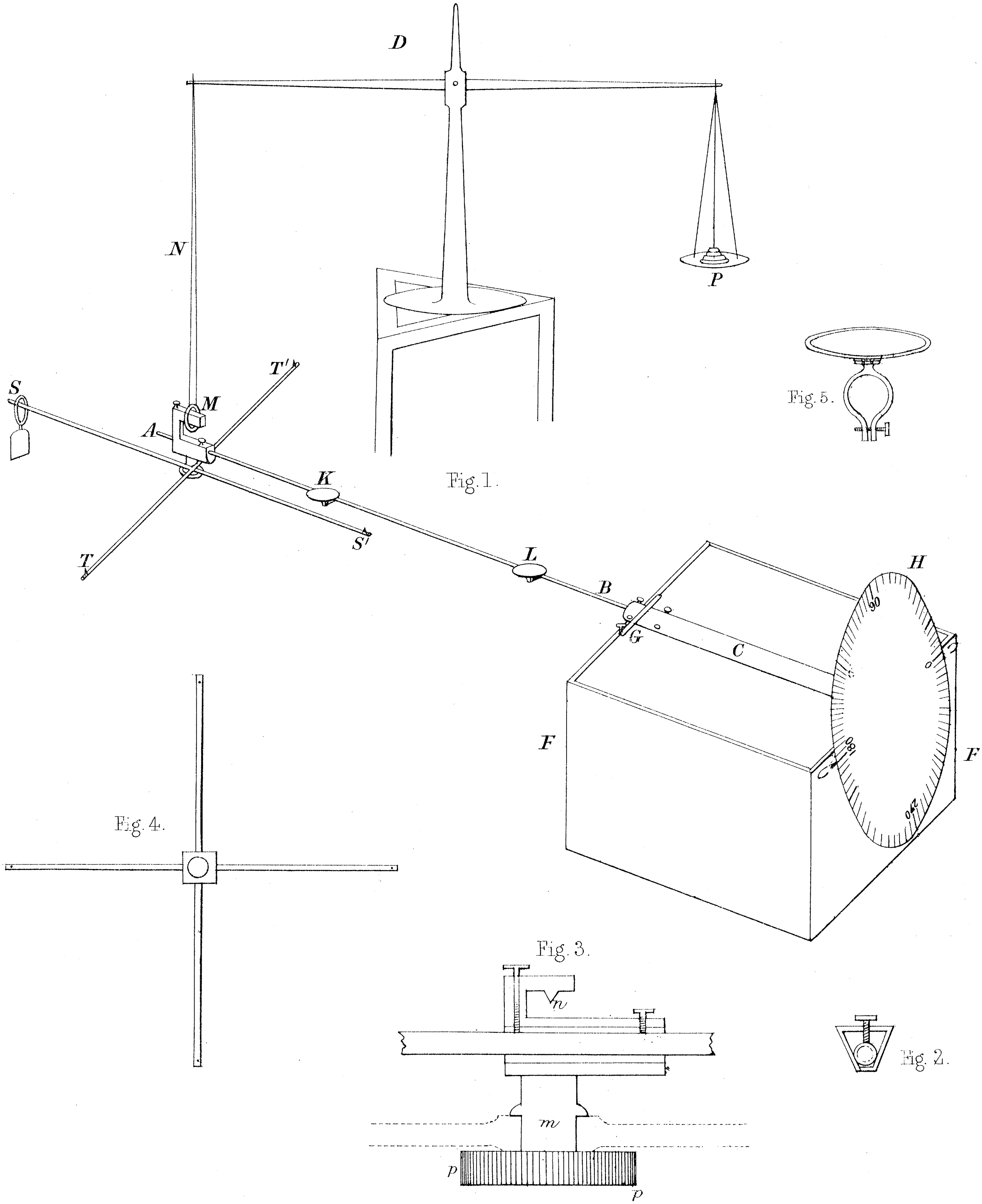

Fig. 1. Fig. 2. Fig. 3. Fig. 4. Fig. 5.

J. Basire lith.